Constraints vs Structures in Type Systems

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1. it exists

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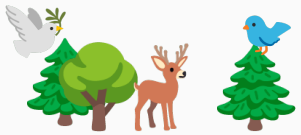
I'd like to convince you that...

1. it exists
2. it's cognitively interesting

Chapter 1: Skirmishes in the Wild 

🫷 Round 1: Rust

Read `'l1: 'l2` as "`'l1` outlives `'l2`."

```
fn first<'a, 'b, 'c, T>(x: &'a T, y: &'b T) ->
&'c T where 'a: 'c {
    let z = &*x; // z : &'d T, implicitly
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Read `&{x} T` as "a reference to `x` lives inside `T`."

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fn first<T>(x: &T, y: &T) -> &{x} T {
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👐 Round 2: Information Flow

Read `!l1 < !l2` as ‘information from `!l1` flows into `!l2`.’

```
let p : !password string = "katya"
let check : 'a string -> 'b bool
    with !password < 'b and 'a < 'b =
    (* returning a bool with info from password *)
    function attempt -> p == attempt
```

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let inv : !alice int -> !alice int =
    fun x -> -x
let suc : !bob int -> !bob int =
    fun x -> x + 1


val inv_suc : 'a int -> 'b int * 'c int
    with !bob < 'c
    and !alice < 'b
    and 'a < 'b, 'c
    and 'a < !alice, !bob
let inv_suc x = (inv x, suc x)
```

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👐 Round 3: Subtyping

Read `'b <: 'a` as ‘`'b` is a subtype of `'a`.’

```
val twice : 'a -> ('a -> 'b) -> 'b where 'b <: 'a
let twice = fun x -> fun f -> f (f x)
```

Read `'b ∨ 'a` as ‘this type is either `'b` or `'a`.’

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val twice : 'a -> ('b ∨ 'a -> 'b) -> 'b
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Read `'b <: 'a` as ‘`'b` is a subtype of `'a`.’

```
val choose : bool -> 'a -> 'c
    where 'a <: 'c and bool <: 'c
let choose = fun b -> fun x ->
    if b then x else false
```

Read `'b ∨ 'a` as ‘this type is either `'b` or `'a`.’

```
val choose : bool -> 'a -> 'a ∨ bool
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```

Chapter 2: A Smoldering Debate 🔥

Generality 🌌 vs Specificity 🌠

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Few constructs that can capture *many* problems *adequately*

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- ❌ Con: More difficult to tell what any particular program fragment does at-a-glance.

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- ❌ Con: More difficult to identify and transfer intuition between problems

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`'a: 'c` `{x}`

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```
'a: 'c
```

```
{x}
```

```
!password < 'b, 'a < 'b
```

```
[ 'a !password ]
```

Generality 🌌 vs Specificity 🌠

`'a: 'c`	`{x}`
`!password < 'b, 'a < 'b`	`[ 'a !password ]`
`a <: c, b <: c`	`a ∨ b`

Generality 🌌 vs Specificity 🌠

```
'a: 'c
```

```
!password < 'b, 'a < 'b
```

```
a <: c, b <: c
```

Inequalities

```
{x}
```

```
[ 'a !password ]
```

```
a ∨ b
```

Domain-Dependent

Implicitness 🤯 vs Explicitness 🤬

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Structure of problem *reified* in programming interface, *explicit intervention* required

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- ❌ Con: Can't easily forget unimportant type structure, more 'fluff' to read and discard

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val both : 'a int -> 'b int -> 'c int
    where 'a < 'c and 'b < 'c
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- The structural approach reifies these properties directly in the syntax via first-class algebraic structures

  structures = **specific** + **explicit**

Note: *Not* a rigorous or prescriptive definition— still descriptive at this point!

Chapter 3: Poking and Prodding 

Experiments

Hypothesis: The structural approach will be easier to use...

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Semantically

For tasks concerned with *distinguished elements* in the problem domain.

- `a <: c, b <: c`
  `a ∨ b`
- `'a -> 'b`
  `[ 'a ] -> [ ]`
- Making *internally represented* information *external* (explicit).

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- Better **generalizability**.
  - Want to come up with design principles for type systems *beyond* Rust or information flow.

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let result : ? bool = send (check p)
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Goal: figure out fuzzy semi-formal *mental models* of type systems that model *misconceptions*.

Chapter 4: A Cliffhanger 🧗

The constraints-structures distinction hints at a rich framework for analyzing the usability of programming languages.

We want to derive general, far-sighted design principles for type systems that place human interface concerns beside mathematical considerations at the foundations of programming languages.

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